Four ways of solving quadratic equations—worked examples
**Method 1 - Solving Graphically**

Solve \( x^2 + 2x - 8 = 0 \)

**Step 1** - Create a table of values to calculate coordinates you can then use to plot the graph of \( y = x^2 + 2x - 8 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 )</td>
<td>25</td>
<td>16</td>
<td>9</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
</tr>
<tr>
<td>( 2x )</td>
<td>-10</td>
<td>-8</td>
<td>-6</td>
<td>-4</td>
<td>-2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>-8</td>
<td>-8</td>
<td>-8</td>
<td>-8</td>
<td>-8</td>
<td>-8</td>
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<td>-8</td>
<td>-8</td>
<td>-8</td>
<td>-8</td>
<td>-8</td>
</tr>
<tr>
<td>( y = x^2 + 2x - 8 )</td>
<td>7</td>
<td>0</td>
<td>-5</td>
<td>-8</td>
<td>-9</td>
<td>-8</td>
<td>-5</td>
<td>0</td>
<td>7</td>
<td>16</td>
<td>27</td>
</tr>
<tr>
<td>( (x,y) )</td>
<td>(-5,7)</td>
<td>(-4,0)</td>
<td>(-3,-5)</td>
<td>(-2,-8)</td>
<td>(-1,-9)</td>
<td>(0,-8)</td>
<td>(1,-5)</td>
<td>(2,0)</td>
<td>(3,7)</td>
<td>(4,16)</td>
<td>(5,27)</td>
</tr>
</tbody>
</table>

**Step 2** - Plot the graph of \( y = x^2 + 2x - 8 \) using the coordinates calculated in your table of values

**Step 3** - Read off the graph the \( x \) values where it crosses the \( x \) axis (the \( y = 0 \) line). These are your solutions

**Solutions** \( x = -4 \) or 2
Method 2 - Solving By Factorising

Solve \( x^2 + 2x - 8 = 0 \)

Step 1 - factorise \( x^2 + 2x - 8 \) by putting it into double brackets

\[ x^2 + 2x - 8 = (x + 4)(x - 2) = 0 \]

Remember, the numbers inside the brackets have to ADD to make 2 and MULTIPLY to make -8

Step 2 - find which values of \( x \) make each bracket equal to zero. These are your solutions

\( (x + 4)(x - 2) = 0 \)

If were \( x = -4 \) we’d have...

\[ (-4 + 4)(-4 - 2) = 0 \]

\[ (0)(-6) = 0 \]

So \( x = -4 \) must be one solution

If were \( x = 2 \) we’d have...

\[ (2 + 4)(2 - 2) = 0 \]

\[ (6)(0) = 0 \]

So \( x = 2 \) must be the other solution

Solutions \( x = -4 \) or \( 2 \)
Method 3- Solving By Using The Quadratic Formula

Solve \( x^2 + 2x - 8 = 0 \)

**Step 1- get the values of a, b and c to use in the formula**

\[ ax^2 + bx + c = 0 \]
\[ x^2 + 2x - 8 = 0 \]

Therefore
\[ a = 1, \ b = 2, \ c = -8 \]

**Step 2- substitute these values for a, b and c into the quadratic formula and go on to simplify and solve for x**

\[ x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a} \]
\[ x = \frac{-2 \pm \sqrt{(2)^2 - ((4)(1)(-8))}}{2(1)} \]
\[ x = \frac{-2 \pm \sqrt{4 - (-32))}}{2} \]
\[ x = \frac{-2 \pm \sqrt{36}}{2} \]
\[ x = \frac{-2 \pm 6}{2} \]
\[ x = \frac{-2 - 6}{2} \quad \text{or} \quad x = \frac{-2 + 6}{2} \]

**Solutions** \( x = -4 \) or \( 2 \)
Method 4- Solving By Completing The Square

Solve \( x^2 + 2x - 8 = 0 \)

**Step 1- find the completed square form of \( x^2 + 2x - 8 \)**

\( x^2 + 2x - 8 \)

Halve the coefficient of \( x \) (which here is 2) and add to \( x \) in a bracket squared

\((x + 1)^2\)

Expand out the bracket

\((x + 1)^2 = x^2 + 2x + 1\)

Subtract the 1 from both sides

\((x + 1)^2 - 1 = x^2 + 2x\)

Now substitute this back into \( x^2 + 2x - 8 \) for the first two terms

\(x^2 + 2x - 8 = (x + 1)^2 - 1 - 8 = 0\)

\((x + 1)^2 - 9 = 0\)

**Step 2- solve this quadratic equation for \( x \)**

\((x + 1)^2 - 9 = 0\)

Add 9 to both sides

\((x + 1)^2 = 9\)

Square root both sides

\(x + 1 = \pm 3\)

Subtract 1 from both sides

\(x = -1 \pm 3\)

\(x = -1 - 3 \quad \text{or} \quad x = -1 + 3\)

**Solutions** \( x = -4 \) or \( 2 \)